Roll No. ----------

M.A. / M.Sc. II Semester Examination, 2014
Statistics
Paper No. STM-204 Statistical inference-II

Time: Three hours M.M.:70

[Write your roll No. at top immediately on the receipt of the question paper]

Note: Attempt any five questions. All questions carry equal marks.

1. **(a)** Define CAN Estimator and state and prove its invariance property.

   (b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a Poisson distribution with mean λ. Show that $\bar{x}$ is CAN for λ. Let $\psi(\lambda) = e^{-\lambda}$ then show that $e^{-\bar{x}}$ is CAN for $e^{-\lambda}$.

2. **(a)** Define best location invariant estimator. Obtain the Pitman estimator of θ when $X \sim N(\theta, 1)$.

   (b) Define Scale Invariant decision rule. Let $x_1, x_2, \ldots, x_n$ be a random sample from a distribution $f(x|\theta)$ with scale parameter θ. If we consider the loss function $L(\theta, a) = \{(a/\theta) - 1\}^2$ then show that the best invariant estimator under this loss function is

$$d(x_1, x_2, \ldots, x_n) = X_n \left[\frac{E\left(x_n / \left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_{n-1}}{x_n} \right) \right)}{E\left(x_n^2 / \left(\frac{x_1}{x_n}, \frac{x_2}{x_n}, \ldots, \frac{x_{n-1}}{x_n} \right) \right)} \right]$$

3. **(a)** Prove that maximum likelihood estimator of parameter θ in one parameter exponential family is CAN for θ.

   (b) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from probability density function $f(x;\theta) = \theta x^{(\theta-1)}$; $0 < x < 1$, $\theta > 0$. Obtain moment estimate of θ and show that it is CAN for θ. Also obtain the moment estimate of θ based on sufficient statistic and work out for its asymptomatic variance.

4. **(a)** Explain confidence set and shortest confidence interval. Obtain $(1-\alpha)$ shortest length confidence interval for σ^2 when $X \sim N(\mu, \sigma^2)$, where μ is known.

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(b) Explain Bayes confidence interval. Let $x_1, x_2, \ldots, x_n$ be a sample from $N(\theta, 1)$, suppose that the prior distribution for θ is $N(0, 1)$. Find the Bayesian confidence interval for the parameter θ.

5. Define most powerful and uniformly most powerful test and **state and prove** Neyman-pearson lemma for randomized test.

6. (a) Let θ be the real parameter and randon variable x has probability density function $f(x; \theta)$, where $f(x; \theta)$ has monotone likelihood ration in $T(x)$. For testing H_0: $\theta \leq \theta_0$ against H_1: $\theta > \theta_0$, show that there exits a UMP given by

$$\emptyset(\mathbf{x}) = \begin{cases} \mathbf{1} & \boldsymbol{if}\ \ \boldsymbol{T(x) > x_0} \\ \boldsymbol{\gamma_i} & \boldsymbol{if}\ \ \boldsymbol{T(x) = x_0} \\ \mathbf{0} & \boldsymbol{if}\ \boldsymbol{T(x) < x_0} \end{cases}$$

where the test has a non decreasing power function and of its size $E_{\theta_0}[\phi(x)] = \alpha$ (provided size is not zero).

(b) Find the M.P. size α test of H_0: $X \sim f_0(x)$, where $f_0(x) = (2\pi)^{-\frac{1}{2}} e^{-\frac{x^2}{2}}$, $-\infty < x < \infty$, against H_1: $X \sim f_1(x)$, $f_1(x) = \frac{1}{2} e^{-|x|}$, $-\infty < x < \infty$, based on a sample of size 1.

7. Explain likelihood ratio test. Let x_i be a random sample of size n_i from $N(\mu_i, \sigma^2)$, $i = 1,2,3, \ldots, k$ where variance of all k normal populations are supposed to be unknown. Find critical region of size α corresponding to likelihood ratio test for testing H_0: $\mu_1 = \mu_2 = \ldots = \mu_k$ against H_1: all μ_i are not equal.

8. Explain Sequential Probability Ratio Test (SPRT) for testing a simple hypothesis against a simple alternative. Construct a sequential probability ratio test for testing the hypothesis H_0: $\sigma^2 = \sigma_0^2$ against H_1: $\sigma^2 = \sigma_1^2$ $(\sigma_1^2 > \sigma_0^2)$ in sampling from a normal distribution $N(\mu, \sigma^2)$. Also obtain its OC and ASN functions.

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